The Universality of Climate Combination Tones

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Nonlinearity in our ear generates Combination tone \( f_1 - f_2 \)

\[
[A \cos(\omega_a t) + B \cos(\omega_E t)]^2 = 0.5 A^2 \cos(2\omega_a t) + AB[\cos((\omega_a - \omega_E)t) + \cos((\omega_a + \omega_E)t)] + 0.5 B^2 \cos(2\omega_E t)
\]
Potential climate nonlinearities?

- Windstress: $|u| \cdot u$
- Convection; soft threshold
- Evaporation: $|u| \cdot dq$
- Moisture transport: $u \cdot \text{grad} \ q$
- Nonlinear advection: $u \cdot \text{grad} \ u$
- SW Cloud feedback: $a \cdot Q$
- Diabatic forcing: $|u| \cdot \Delta q$

$u = u_m + u_a + u_e$
$q = q_m + q_a + q_e$

$\Rightarrow u_a \cdot q_e \ or \ u_e \cdot q_a$; combination tones and self interaction terms through $u_e \cdot q_e$
Applications to climate dynamics

The role of combination tones in

- ENSO termination
- Asian Monsoon systems
- Sea level shifts
- Westerlies
- Sea-ice
- Ice ages
- etc
ENSO and its interactions

PDO, IPO

Annual cycle

Combination Tones

WWBs

ENSO
ENSO and its interactions

Greenhouse warming

组合音的

PDO, IPO

Annual cycle

WWBs

ENSO

Combination Tones
ENSO-WWB interaction (Multiplicative Noise)

Interannual ENSO mode (Recharge physics)

Termination of El Nino events after DJF

Seasonally modulated wind anomalies

Annual cycle in western tropical Pacific

2:1 Phase locking

Seasonal variance modulation

Combination Tones of EOF modes

Philippine Anticyclone/ SE Asian Monsoon

Zonal SPCZ events

Taimasa events
Interannual ENSO mode (Recharge physics) 

Annual cycle in western tropical Pacific

Stuecker, Timmermann, Jin, Ren (2013) Nature Geoscience
Where does EOF2 come from?

A hint is provided by the power spectrum of EOF2. It shows **nonlinear combination tones** at 1-f and 1+f with f being the ENSO frequency.

EOF mode 2 emerges from the nonlinear interaction between ENSO and annual cycle.

\[
[A \cos(\omega_a t) + B \cos(\omega_0 t)]^2 \\
= 0.5 A^2 \cos(2\omega_a t) + AB[\cos((\omega_a - \omega_0) t) + \cos((\omega_a + \omega_0) t)] + 0.5 B^2 \cos(2\omega_0 t)
\]

PC2 (wind-shift) = PC1 x cos (\omega_a t) = ENSO x annual cycle
The importance of the Combination tone for the El Niño termination?

McGregor, Timmermann, Schneider, Jin, JCL (2012)

Southward shift of El Niño-related wind anomaly weakens downwelling Kelvin wave

PC2 – related wind stress curl accelerates discharge in a meridionally asymmetric way.

Equatorial zonal mean thermocline depth

Effect of PC1 winds (recharge)

Effect of PC2 winds
Parametrically forced oscillator

EOF2 effect parameterized as

\[ \lambda_0 \cos(\omega_a t) T \]

\[ \lambda(t) = \lambda_0 \cos(\omega_A t) \]

\[
\frac{dT}{dt} = -\lambda(t)T + \omega_0 h, \\
\frac{dh}{dt} = -\omega_0 T, \\
T \approx C \left[ \cos(\omega_0 t) + \frac{\lambda_0}{\omega_A^2 - 4\omega_0^2} \left\{ \frac{\omega_0 \cos(\omega_A t) \sin(\omega_0 t)}{\omega_A^2 - 2\omega_0^2} - \frac{\omega_A^2 - 2\omega_0^2}{\omega_A} \sin(\omega_A t) \cos(\omega_0 t) \right\} \right],
\]

ENSO/annual cycle interactions
Impacts: The ENSO/Monsoon bridge is a combination tone

Stuecker et al. 2014, JCL, submitted

Philippine Sea anticyclone
Combination Tone dynamics of the Philippine Sea Anticyclone

Stuecker et al. 2014, JCL, submitted

CM 2.1 ensemble
Combination mode impacts on sea level

Widlansky et al. 2014 J Climate
Very low sea levels, or ‘taimasa’, affect tropical western Pacific islands during strong El Niño.

Prolonged low sea levels in Southwest Pacific ($r = 0.60$ at lag 6 months)
Impacts: El Nino ‘Taimasa’ events

Sea level low stands in SW tropical Pacific are a result of the atmospheric Combination tone.

Widlansky et al. 2014 J Climate
Summary

• Combination Tones (Modes) are a simple nonlinear concept that allows the study of seasonally modulated interannual signals.
• They provide explanation for near annual variability
• Seasonal forecasts are strongly determined by combination tone dynamics
• How well are Combination tones captured in models?
• Simple ENSO-bandpass filtering removes an essential part of ENSO dynamics